# Vignette for MultiMed package 

Simina M. Boca<br>Innovation Center for Biomedical Informatics and<br>Department of Oncology, Georgetown University Medical Center email: smb310@georgetown.edu,<br>Joshua N. Sampson<br>Biostatistics Branch, Division of Cancer Epidemiology and Genetics, National Cancer Institute<br>email: joshua.sampson@nih.gov

June 15, 2024

## 1 Overview

The MultiMed package implements a permutation method which adjusts for "multiple comparisons" when testing whether multiple biomarkers are mediators between a known risk factor and a disease. The approach is described in the companion paper [Boca et al., 2014], "Testing multiple biological mediators simultaneously." This method can significantly improve the power to detect mediators over the standard Bonferroni correction.

We first need to load the package:
> library(MultiMed)

## 2 Performing the test of mediation

The scenarios which can be considered are shown in Figure 1 for the single mediator case and Figure 2 (also shown in the [Boca et al. 2014] paper) for the multiple mediator case. Here, we consider simulating data where the exposure $E$, the mediator(s) $M$ (or $M_{i}, i=1, \ldots, K$ ), and the outcome $Y$ are normally distributed. We denote by $\sigma_{E}^{2}$ the variance of $E$, by $\sigma_{M}^{2}\left(\sigma_{M_{i}}^{2}\right)$ the variance of $M\left(M_{i}\right)$ conditional on $E$, and by $\sigma_{Y}^{2}$ the variance of $Y$ conditional on $E$ and $M\left(M_{i}\right)$.

Figure 1: A scenario with a single possible mediator between exposure and outcome.


### 2.1 The medTest function

The function used to perform the test of mediation is medTest. It has seven arguments: $E, M, Y, Z, n p e r m$, W , and useWeightsZ. E, M, and Y represent matrices of size $n \times 1, n \times K$, and $n \times 1$, respectively, giving the exposure, mediator, and outcome values, where $n$ is the sample size and $K$ is the number of mediators.
$E$ and $Y$ can also be inputted as vectors. The $Z$ argument is either NULL or a numerical matrix having $n$ rows. If it is not NULL, then the exposure, mediators, and outcome will all be initially regressed on Z , with the residuals being used in the mediation analysis. The nperm argument gives the number of permutations used to estimate the null distribution, the default being 100. The w argument specifies whether any weighting should be done for the $E-M$ association, as would be needed, for instance, in a scenario which considers a case-control study. The default is $w=1$, which means that all the study participants are equally weighted; w may also be given as a vector of length $n$, in which case it is first standardized to sum to 1 . The useWeightsZ argument can be TRUE, in which case the weights in $w$ are used for the initial regression on $Z$, or FALSE, in which case equal weights are used for this initial step.

### 2.2 Simulated example: Single mediator case

For a sample size of $n=100$, we can simulate a dataset with a single mediator in the following way:

```
> set.seed(20183)
> alpha <- 0.2
> beta <- 0.2
> gamma <- 0.4
> n <- 100
> sigma2E <- 1
> sigma2M <- 1 - alpha^2
> sigma2Y <- 1 - beta^2 * (1 - alpha^2) - (alpha * beta + gamma)^2
> ## exposure:
> E <- rnorm(n, O, sd = sqrt(sigma2E))
> ## mediator:
> M <- matrix(0, nrow = n, ncol = 1)
> M[, 1] <- rnorm(n, alpha * E, sd = sqrt(sigma2M))
> ## outcome:
> Y <- rep(0, n)
> for (subj in 1:n) Y[subj] <- rnorm(1, beta * M[subj, ], sd = sqrt(sigma2Y))
```

Note that the values of $\sigma_{E}^{2}, \sigma_{M}^{2}$, and $\sigma_{Y}^{2}$ were chosen so that the marginal variances of $E, M$, and $Y$ are 1 .
To perform a test of mediation, we use the medTest function. The output is a matrix with two columns: S , the test statistic used (the absolute value of the product of the correlations between $E$ and $M$ and between $r_{M \mid E}$ and $r_{Y \mid E}$, where $r_{Z_{1} \mid Z_{2}}$ represents the residual obtained from regressing $Z_{1}$ on $Z_{2}$ ) and $p$, the $p$-value:

```
> medTest(E, M, Y, nperm = 500)
```

    S p
    [1,] 0.013229640 .53

### 2.3 Simulated example: Multiple mediator case

Now consider a scenario with $K=10$ mediators and a sample size of $n=100$.

```
> set.seed(380184)
> alpha <- c(rep(0, 6), rep(0.3, 2), rep(0, 2))
> beta <- c(rep(0, 6), rep(0, 2), rep(0.3, 2))
> gamma <- 0.6
> alpha
[1] 0.0 0.0 0.0 0.0 0.0 0.0 0.3 0.3 0.0 0.0
> beta
[1] 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.3 0.3
```

Figure 2: A scenario with $K$ possible mediators between exposure and outcome.


```
> n <- 100
> sigma2E <- 1
> sigma2M <- 1-alpha^2
> sigma2Y <- 1-sum(beta^2*sigma2M)-(sum(alpha*beta)+gamma)^2
> sigma2M
```

[1] $1.001 .001 .001 .001 .001 .00 \quad 0.91 \quad 0.911 .001 .00$

```
> sigma2Y
```

[1] 0.46
Note that in this case alpha and beta are vectors having the $i^{\text {th }}$ elements be $\alpha_{i}$, respectively $\beta_{i}$, where $i=1, \ldots, 10$ indexes the mediators. Similarly, sigma2M is a vector, with the $i^{\text {th }}$ element being $\sigma_{M_{i}}^{2}$. The values of $\sigma_{E}^{2}, \sigma_{M_{i}}^{2}$ and $\sigma_{Y}^{2}$ were chosen so that the marginal variances of $E, M_{i}, Y$ are 1 .

We first simulate the data:

```
K <- length(alpha)
> ## exposure:
> E <- rnorm(n, 0, sd = sqrt(sigma2E))
> ## mediator:
> <- matrix(0, nrow = n, ncol = K)
> for (i in 1:K) {
+ M[, i] <- rnorm(n, alpha[i] *E, sd = sqrt(sigma2M[i]))
+ }
> ## outcome:
> Y <- rep(0, n)
> for (subj in 1:n)
+ Y[subj] <- rnorm(1, sum(beta*M[subj,])+gamma*E[subj], sd=sqrt(sigma2Y))
```

We then use the medTest once again to perform the test of mediation. The output is now a matrix with 10 rows, each row giving the test statistic $S$ and the $p$-value $p$ for each mediator. Note that the $p$-values are already implicitly considering the multiple tests being performed, so no further adjustment is necessary:

```
> medTest(E, M, Y, nperm = 500)
    S p
[1,] 0.0115085655 1.000
[2,] 0.0008037094 1.000
```

```
    [3,] 0.0009221887 1.000
    [4,] 0.0161794377 0.998
    [5,] 0.0016529532 1.000
    [6,] 0.0001764986 1.000
    [7,] 0.0343911724 0.774
    [8,] 0.0554955400 0.296
    [9,] 0.0031333508 1.000
[10,] 0.0447346023 0.534
```


### 2.4 Data analysis: Metabolites as mediators

We consider a data example from the [Boca et al., 2014] paper, using the Navy Colorectal Adenoma casecontrol study [Sinha et al. 1999], with daily fish intake as the exposure of interest $E$ and colorectal adenoma status as the outcome $Y$. The possible mediators are 149 serum metabolites, whose values were previously batch normalized and log transformed.

We first load the dataset:
> data(NavyAdenoma)
The first 5 columns of the NavyAdenoma object represent: daily fish intake, BMI, gender (coded as 0 for male, 1 for female), age, and current smoking status (coded as 0 for non-smoker, 1 for current smoker):

```
> colnames(NavyAdenoma) [1:5]
```

[1] "Fish" "BMI" "Female" "Age" "Smoking"

The next 149 columns represent the metabolite values, while the last column represents the case-control status:

```
> colnames(NavyAdenoma)[c(6:9,154)]
[1] "glycine" "serine" "betaine" "alanine" "erythritol"
> colnames(NavyAdenoma) [155]
[1] "Adenoma"
> table(NavyAdenoma$Adenoma)
    0}
1 2 9 1 2 9
```

Due to the retrospective sampling, we consider weights incorporating the prevalence of adenoma in this age category (approximately 0.228 ) and the fraction of cases in the dataset for the E-M associations:

```
> prev <- 0.228
> p <- sum(NavyAdenoma$Adenoma==1)/nrow(NavyAdenoma)
> p
```

[1] 0.5

```
> w <- rep(NA, nrow(NavyAdenoma))
> w[NavyAdenoma$Adenoma == 1] <- prev/p
> w[NavyAdenoma$Adenoma == 0] <- (1-prev)/(1-p)
> table(w)
w
0.456 1.544
    129 129
```

We use medTest to perform the test of mediation, adjusting for the covariates BMI, gender, age, and current smoking status. As in the Boca et al. [2014] paper, we perform this adjustment using equal weights, rather than using the weights in w , but users can consider using the weights in w both here and downstream:

```
> set.seed(840218)
> medsFish <- medTest(E=NavyAdenoma$Fish,
+ M=NavyAdenoma[, 6:154],
+ Y=NavyAdenoma$Adenoma,
+ Z=NavyAdenoma[, 2:5],
+ nperm=1000, w=w,
+ useWeightsZ=FALSE)
```

Now find metabolite which has the lowest p-values:

```
> rownames(medsFish) <- colnames(NavyAdenoma[,-c(1:5, 154)])
> medsFish[which.min(medsFish[,"p"]),,drop=FALSE]
    S p
docosahexaenoate (DHA; 22:6n3) 0.04989712 0.051
```

Thus, we conclude that DHA (fish oil) is a possible mediator of the association between fish intake and colorectal adenoma.

## References

S. M. Boca, R. Sinha, A. J. Cross, S. C. Moore, and J. N. Sampson. Testing multiple biological mediators simultaneously. Bioinformatics, 30(2):214-220, 2014.
R. Sinha, W. H. Chow, M. Kulldorff, J. Denobile, J. Butler, M. Garcia-Closas, R. Weil, R. N. Hoover, and N. Rothman. Well-done, grilled red meat increases the risk of colorectal adenomas. Cancer Research, 59 (17):4320-4324, 1999.

